ingly rapid rates of attenuation, remains to be determined. With the help of Dr. H. L. Howes, to whose assistance throughout this investigation the author is greatly indebted, this question is now under investigation.

[1] Lenard, *Ann. Physik*, ser. 4, **31**, 641 (1910).
[2] E. L. Nichols, Paper presented before the Amer. Philos. Soc., April, 1916.
[3] Nichols and Howes, Paper presented before the Amer. Physic. Soc., April, 1916.
[4] Nichols and Howes, *Physic. Rev.*, ser. 2, **4**, 19 (1914).
[5] Kennard, *Physic. Rev.*, ser. 2, **4**, 278.

THE PYRANOMETER: AN INSTRUMENT FOR MEASURING SKY RADIATION

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This instrument, as its name (from the Greek πῦρ, fire, ἀνά, up, μέτρον, a measure) indicates, is intended to measure the heat equivalent of radiation received from or going out toward the complete hemisphere above the plane of the measuring surface. We have devised two satisfactory types of the instrument, both derived in principle from the electrical compensation radiation instruments of the late K. Ångström. The full description of the instruments and tests of them will be found in a paper now being published in the *Smithsonian Institution Miscellaneous Collections*. The instruments are adapted to measure direct solar radiation, the total radiation of the sun and sky combined, that of the sky alone, and nocturnal radiation. It is possible to employ screens of selective transmission and thus to limit the measurements to selected spectrum regions. The instruments are of primary standard type, but have been compared with the standardized pyrheliometers of the Smithsonian Institution and found accordant. No auxiliary apparatus other than that employed with the Ångström pyrheliometer is required, and the observations are easy to make.

The simpler form of pyranometer comprises a single blackened manganin strip, 3 mm. wide, 6 mm. long, placed centrally in the plane of the upper surface of a nickel-plated copper disk 75 mm. in diameter. Copper blocks insulated from the rest of the disk, but continuous with it in surface, serve to connect the insulated manganin strip with an electric heating current of adjustable strength. A sensitive thermo-electric couple fastened by means of thin waxed paper to the rear surface of the manganin strip, and embedded at the other end in a recess of the copper disk serves to indicate changes of temperature of the strip. Concen-

tric with the strip is a hollow hemispherical screen of ultra-violet crown glass, 26 mm. in outer diameter and 2 mm. thick. Its purpose is to admit rays of shorter wave-lengths, such as form essentially the whole strength of the direct and scattered solar rays, but to cut off rays of great wave-length proper to the emission of a body at ordinary temperatures. During measurements of nocturnal radiation this glass screen is removed. A nickel-plated hemispherical shell of polished nickel-plated copper encloses this glass, and is removable at pleasure.

If now the shutter is opened diffuse sky-radiation falls upon the strip and warms it, producing a deflection of a moving-coil galvanometer in the circuit of the thermo-couple. The shutter being then closed, an equal deflection may be produced by the electric heating current. As corrected to allow for losses by reflection of the glass and the imperfect absorption by lampblack, the energy dissipated in the strip by the heating current measures the energy of radiation. As constructed the sensitiveness of this instrument is so great that it proves convenient to balance the deflection to zero by means of a potentiometer current in the galvanometer circuit, and so to reduce the operations to the zero method. A defect of this simple form of pyranometer is found to be caused by the slow warming of the glass-covered portion of the copper disk when the shutter is opened, which at other times shades that area of the surface. This warming induces a secondary deflection, because it affects the two differently situated ends of the thermo-couple differently. Experiments have shown, however, that practically the full deflection due to direct heating of the strip occurs in 20 seconds, and that the secondary deflection begins to be sensible after 20 seconds. Accordingly the error is eliminated by balancing the primary deflection by the potentiometer current after exactly 20 seconds, then closing the shutter and waiting two minutes for the secondary heating to subside, before adjusting the heating current.

Fearing that this defect might prove more serious in nocturnal radiation work, we devised a second form of pyranometer. In this form there are *two* blackened manganin strips side by side, each 2 mm. wide, 6 mm. long, separated by a nickel-plated copper bar 2 mm. wide, and both insulated as in the simple form by vertical mica strips coming exactly to the surface of the plate. Thermo-couples connect the two strips at the back, the hot junction behind one strip, the cold junction behind the other. As the two strips absorb radiation equally, there would be an equal rise of temperature, if it were not that one strip is 10 times as thick as the other. Owing to this the conduction to the ends is so much greater for the thick strip that a difference of temperature arises, and a deflec-

tion of the galvanometer ensues. The heating current is divided between the two strips, and by suitable resistance coils the circuit is adjusted once for all so that whatever the strength of the heating current it produces equal dissipation of energy in the two strips. If now after closing the shutter the heating current is graduated until the deflection formerly produced by radiation is reproduced by electrical heating, the energy dissipated in either strip is the measure of the absorbed radiation. In the two strip pyranometer the secondary deflection by indirect heating is unimportant, because of the symmetry of the arrangement. However, to avoid this source of error altogether the exposure is limited to 30 seconds, and a full minute is allowed to lapse before introducing electric heating.

Numerous measurements of the sky-radiation have been made from the North Tower of the Smithsonian Institution. On fine days the sky-radiation alone received on a horizontal surface ranges from 0.07 to 0.13 calories per square centimeter per minute. On cloudy days, not thick enough for rain, the values run from 0.20 to 0.30 calories according to the kind of cloudiness prevailing. Measurements were made on the reflection from new fallen snow, and for total solar and sky radiation this proved to be 70%.

In the simpler form the instrument is so sensitive that it could be used in the deep shade of a forest, or with screens of selective transmission, so that it would be suited to botanical as well as meteorological investigations. As in the case of the silver disk pyrheliometer, the Smithsonian Institution may undertake to prepare pyranometers at cost (approximately $150) where valuable investigations may be promoted thereby.

NOTE ON LUCAS' THEOREM

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In a recent note[1] I ventured to give a proof, which I thought might be new, of Lucas' Theorem and one of its more immediate generalizations to rational functions. Professor Bôcher has kindly called my attention to the fact that the same proof had previously been given by him[2] and that the extension to rational functions was a special case (in the method of proof as well as in the results obtained) of other of his results.[3] I called attention to the interesting fact that this proof of Bôcher's applies without modification at once to integral functions of class zero. It is the purpose of this second note to show that it also applies without modi-